VARIABLES, TERMS, AND EXPRESSIONS

Mathematics has developed a language to itself in order to clarify concepts and remove ambiguity from the analysis of problems. To achieve this there are some basic definitions to allow us to speak the same language. This review emphasizes concepts learned in Algebra I which are essential skills as you begin Algebra II.

Variable: A quantity that is represented by a letter or symbol that is unknown, unspecified, or can change within the context of a problem.

Expression: A combination of terms using addition and subtraction.

Ex 1: $x + 3$ (x is the variable) Ex 2: $2\alpha - 1$ (α is the variable) Ex 3: $\sin\theta + 2$ (θ is the variable)

Circle the variables in the following expressions.

1. $3x - 2$
2. $z^2 + 4z + 4$
3. $2\cos\beta - 5$
4. 6^y

Terms: A single number or combination of numbers and variables using exclusively multiplication or division. This definition will expand when we introduce higher-level functions.

Monomial – one term **Binomial** – two terms **Trinomial** – three terms **Polynomial** – many terms

Ex 1: $7xy^3$	Ex 2: $3x + 4y$	Ex 3: $8x^3 - 9x - 7$	Ex 4: $5y^2z + 10xy - 9xz^6 + 6$
Monomial	**Binomial**	**Trinomial**	**Polynomial**

Classify each polynomial as a monomial, a binomial, or a trinomial.

5. $-0.7ab^5$ __________
6. $5 + 3x - 6x^3$ __________
7. $2pm - 5p^2n$ __________
8. $\frac{8x}{-2}$ __________

Practice Problems: *Simplify each of the following expressions by combining like terms. Be careful to only combine terms that have the same variables and powers.*

9. $2x^2 + 8x - 1 + 5x^2 - 2x - 8$

10. $-5x^2 - 2x + 10 - x^2 + 7x + 5$

11. $4x^2y - 2xy^2 + 9xy^2 - x^2y$

12. $7x^3 - 2x^2y + 4xy^2 - y^3 + 2x^2 + 9x^2y + 4y^3$

13. Given the algebraic expression $\frac{12x+12}{x^2-1}$ do the following:

a) Evaluate the expression for when x = 7.

b) Evaluate the expression for when x = 4.

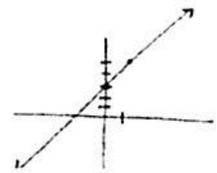
14. Which of the following is equivalent to the expression $2(x - 6) + 4(2x + 1) + 3$? (*You must show work for any credit to be given.)*

(a) $8(x - 2)$ (b) $5(2x - 1)$ (c) $4(2x + 3)$ (d) $10(x - 1)$

LINEAR EQUATIONS & INEQUALITIES

<u>Recall</u>: **Linear Equations** are equations where the variable is only raised to the first power. They all can be written in the form $y = mx + b$, where m is the slope and b is the y-intercept. Graphs of linear functions are lines.

<u>Ex</u>: $y = 2x + 3$ slope = 2 y-intercept = 3 (0,3)



<u>Ex</u>: *Solve each of the following linear equations for x:*

(a) $3x + 5 = 26$
$3x = 21$
$\boxed{x = 7}$

(b) $8x - 7 = 4x - 5$
$4x = 2$
$\boxed{x = \frac{2}{4} = \frac{1}{2}}$

Note: Always reduce fractions to <u>lowest terms</u>

(c) $6(x+4) - 2(x-1) = 2x + 20$
$6x + 24 - 2x + 2 = 2x + 20$
$4x + 26 = 2x + 20$
$2x = -6$
$\boxed{x = -3}$

(d) $\frac{6x+8}{2} = -6$
$6x + 8 = -12$
$6x = -20$
$\boxed{x = -\frac{20}{6} = -\frac{10}{3}}$

<u>**Linear Inequalities**</u> contain either *less than*, <, *greater than*, >, *less than or equal to*, ≤, or *greater than or equal to*, ≥, signs. Linear inequalities are solved much like linear equations, but remember these additional rules:

- When multiplying by a negative number, you must switch the inequality sign
- Answers should be written in <u>**interval notation**</u>

<u>Ex</u>: $x \le 2$ in interval notation is written $(-\infty, 2]$
$x > -3$ in interval notation is written $(-3, \infty)$
$-5 \le x < 3$ in interval notation is written $[-5, 3)$

<u>Ex</u>: *Solve the following linear inequalities and express your answers in interval notation:*

(a) $2x + 6 \le -3x + 1$
$5x \le -5$
$x \le -1$
$\boxed{(-\infty, -1]}$

(b) $7 - 3x < 4 + 8x$
$-11x < -3$
$x > \frac{3}{11}$

Note: Divide by a negative, so switch inequality sign

$\boxed{\left(\frac{3}{11}, \infty\right)}$

Practice Problems: *Solve each of the following linear equations. Reduce any non-integer answers to fractions in simplest form.*

(1) $7x+5=2x-35$

(2) $3(y-1)+2=y+9$

(3) $5(2a-6)+2(4a+3)=8a-9$

(4) $\dfrac{5(x-3)}{2}-1=14$

(5) $18-2(s+7)=\dfrac{8s-20}{2}-2$

Solve the following linear inequalities. Write solutions in interval notation:

(6) $4x+3\geq 2x-9$

(7) $3(x-7)+2<5x-14$

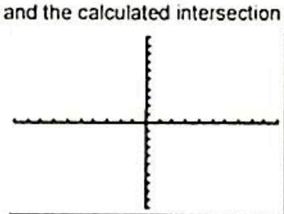
(8) $-\dfrac{x}{3}-7\geq -4$

Answer the following problem:

(9) When finding the intersection of two lines algebraically, you first set the two equations equal to each other. Find the intersection point of the lines $y=5x+1$ and $y=2x+4$. (*First set the equations equal and solve the corresponding equation to find the x-coordinate of the intersection, then plug your answer back in to either of the original equations to find the corresponding y-coordinate.)*

(10) Confirm your answer to question (9) above by using a graphing calculator and following steps:

- Hit Y= and enter the two equations from above as y_1 and y_2
- Go to ZOOM 6 (Standard) to view the two graphs
- Go to 2nd TRACE 5 (Intersect). When the calculator says "First Curve," use the left and right arrows to move the cursor onto the intersection point. Hit ENTER. When it says "Second Curve", hit ENTER again, and when it says "Guess", hit ENTER a third time. The calculated x and y values should match your answer to question (9) above.
- Sketch the graph shown on your calculator and the calculated intersection point on the graph below.



COMMON ALGEBRAIC TECHNIQUES

__Part I:__ PEMDAS is a short way to remember the correct order to evaluate expressions involving multiple operations:

P: Operations in Parentheses
E: Exponents
M/D: Multiply and Divide (in order from left to right)
A/S: Add and Subtract (in order from left to right)

Evaluate the following expressions ***without the use of a graphing calculator*** *using the using the correct order of operations: (SHOW ALL WORK for credit to be given.)*

1) $8+16\div4-5-3$

2) $3\left[(7-5)^2+(20-19)^2\right]+14$

3) Evaluate $-9x-7x^2-1$ when $x=7$.

4) Evaluate $-2\sqrt{x^2+36}$ when $x=-8$.

5) If $x=5$ and $y=-2$, evaluate $\dfrac{x+y}{x^2-y^2}$.

6) Evaluate $\sqrt{\dfrac{|x-8|}{5x^2+4}}$ if $x=2$ and simplify.

__Part II:__ The zeros of an expression are all values of *x* whose corresponding *y*-values are equal to 0.

__Ex__: $x=3$ is a zero of the quadratic expression $4x^2-13x+3$ because plugging in $x=3$ gives:

$$\begin{aligned}&4(3)^2-13(3)+3\\=&4(9)-13(3)+3\\=&36-39+3\\=&0\end{aligned}$$

We can find the zeros of a polynomial by:

- Setting the equation equal to zero and solving (possibly by factoring)
- Graphing the equation and determining at which *x*-values the graph intersects the *x*-axis.
- Going to the TABLE feature in your calculator and looking for the *x*-values that result in a 0 for *y*.

7) Which of the following is a complete list of zeros for the polynomial $12x^2-37x-10$? (*You must show all work for credit to be given.)*

(a) $\dfrac{3}{10}$ and -4 (b) $\dfrac{3}{10}$ and $-\dfrac{1}{4}$ (c) $\dfrac{10}{3}$ and $-\dfrac{1}{4}$ (d) $-\dfrac{3}{10}$ and -4

8) Which of the following is a complete list of zeros for the polynomial $2x^3 + 6x^2 + 4x$? (*You must show all work for credit to be given.)*

(a) 0, 1, and 2 (b) –2, –1, and 0 (c) –1 and 2 (d) 1 and 2

Part III: When **simplifying radicals,** express the radicand as a product of two factors, where one of the factors is the greatest perfect square less than or equal to the radicand:

Ex: $\sqrt{98} = \sqrt{49 \cdot 2}$
$\sqrt{49} \cdot \sqrt{2}$
$\boxed{7\sqrt{2}}$

Ex: $2\sqrt{54} = 2\sqrt{9 \cdot 6}$
$2\sqrt{9} \cdot \sqrt{6}$
$\boxed{6\sqrt{6}}$

Ex: $\sqrt{128} = \sqrt{64 \cdot 2}$
$\sqrt{64} \cdot \sqrt{2}$
$\boxed{8\sqrt{2}}$

Simplify the following expressions involving radicals completely:

9) $\sqrt{32}$ 10) $5\sqrt{125}$ 11) $-3\sqrt{27}$ 12) $4 + 2\sqrt{12}$

13) When solving the equation $6x^2 - 2x - 3 = 0$, Jim uses the quadratic formula and gets $\dfrac{2 \pm \sqrt{76}}{12}$ as his solutions. What would his answers look like in simplest radical form?

Part IV: Graphing Points in the *x*-*y* Coordinate Plane

14) Assume $a > 0$. State which quadrant each of the following coordinates would lie in:

$(-a, -a)$ $(-a, a)$ $(a, -a)$ (a, a)

__________ __________ __________ __________

BASIC EXPONENT MANIPULATION

When simplifying expressions involving exponents, remember these basic properties:

$x^a \cdot x^b = x^{a+b}$	$EX:\ 5^2 \cdot 5^3 = 5^5$
$\frac{x^a}{x^b} = x^{a-b}$	$EX:\ \frac{7^4}{7^1} = 7^3$
$(x^a)^b = x^{ab}$	$EX:\ (2^3)^4 = 2^{12}$
$x^0 = 1$	$EX:\ 742^0 = 1$
$x^{-a} = \frac{1}{x^a}$	$EX:\ 8^{-2} = \frac{1}{8^2}$
	$EX:\ \frac{1}{3^{-2}} = 3^2$

<u>Ex</u>: Simplify the following expressions using only positive exponents.

(a) $\frac{(10w^3)^2}{5w} = \frac{10^2w^6}{5w} = \frac{100w^6}{5w} = \boxed{20w^5}$

(b) $(2x^3)(6x^5) = \boxed{12x^8}$

==

Apply exponent rules to answer each of the following multiple choice questions. You must SHOW ALL WORK for credit to be given.

1. What is the value of 3^{-2}?

(a) $\frac{1}{9}$ (b) $-\frac{1}{9}$ (c) 9 (d) -9

2. What is the value of $3^0 + 3^{-2}$?

(a) 0 (b) $\frac{1}{9}$ (c) $\frac{10}{9}$ (d) 6

3. The product of $2x^3$ and $6x^5$ is

(a) $10x^8$ (b) $12x^8$ (c) $10x^{15}$ (d) $12x^{15}$

4. The expression $2^3 \cdot 4^4$ is equivalent to

(a) 8^{12} (b) 8^7 (c) 2^{11} (d) 2^9

5. If $f(x) = 4x^0 + (4x)^{-1}$, what is the value of $f(4)$?

(a) -12 (b) 0 (c) $\frac{17}{16}$ (d) $\frac{65}{16}$

Simplify each of the following expressions using only positive exponents.

6. $(6x^3y^6)^2$

7. $\frac{-32x^8}{4x^2}$

8. $\frac{(4x^3)^2}{2x}$

9. $(3x^2y^4)(4xy^2)$

MULTIPLYING POLYNOMIALS

Recall: The **Distributive Property** states $a(b+c) = ab + ac$. We can extend this property to help us multiply monomials by a polynomial:

Ex 1: $6x(x^2 - 4x + 8) =$ $6x(x^2) - 6x(4x) + 6x(8)$

$\boxed{6x^3 - 24x^2 + 48x}$

When multiplying polynomials by polynomials, we have to use the distributive property multiple times and combine like terms.

Ex 2: (Binomial)(Binomial)

$(3x+2)(2x+5) =$ $3x(2x+5) + 2(2x+5)$

$6x^2 + 15x + 4x + 10$

$\boxed{6x^2 + 19x + 10}$

We can use the Box Method to multiply:

	$2x$	5
$3x$	$6x^2$	$15x$
2	$4x$	10

$\boxed{6x^2 + 19x + 10}$

Another shortcut for multiplying a binomial by a binomial is called **FOIL**:

$(3x+2)(2x+5)$

First terms	→	$3x \cdot 2x$	→	$6x^2$
Outside terms	→	$3x \cdot 5$	→	$15x$
Inside terms	→	$2 \cdot 2x$	→	$4x$
Last terms	→	$2 \cdot 5$	→	$+10$

$\boxed{6x^2 + 19x + 10}$

The distributive property can be extended to multiply binomials by trinomials, trinomials by trinomials, etc... Here are a few examples:

Ex 3: $(4x+3)(2x^2 - 5x - 3) =$ $4x(2x^2 - 5x - 3) + 3(2x^2 - 5x - 3)$

$8x^3 - 20x^2 - 12x + 6x^2 - 15x - 9$

$\boxed{8x^3 - 14x^2 - 27x - 9}$

Ex 4: $\left(2x^2 + \frac{7}{3}x + \frac{3}{5}\right)\left(\frac{1}{2}x^2 - 3x + 7\right) =$ $2x^2\left(\frac{1}{2}x^2 - 3x + 7\right) + \frac{7}{3}x\left(\frac{1}{2}x^2 - 3x + 7\right) + \frac{3}{5}\left(\frac{1}{2}x^2 - 3x + 7\right)$

$x^4 - 6x^3 + 14x^2 + \frac{7}{6}x^3 - 7x^2 + \frac{49}{3}x + \frac{3}{10}x^2 - \frac{9}{5}x + \frac{21}{5}$

$\boxed{x^4 - \frac{29}{6}x^3 + \frac{73}{10}x^2 + \frac{218}{15}x + \frac{21}{5}}$

Ex 5: $(x-2)(x+4)(x-7) =$ $(x^2 + 4x - 2x - 8)(x-7)$ ← *First multiply* $(x-2)(x+4)$

$(x^2 + 2x - 8)(x-7)$ ← *Now multiply* $(x^2 + 2x - 8)(x-7)$ *using the distributive property three times.*

$x^2(x-7) + 2x(x-7) - 8(x-7)$

$x^3 - 7x^2 + 2x^2 - 14x - 8x + 56$

$\boxed{x^3 - 5x^2 - 22x + 56}$

Practice: *Evaluate each of the following expressions.*

1. $(x+5)(x+8)$

2. $(5x-2)(2x-3)$

3. $(2x^3+1)(5x^3+4)$

4. $\left(\frac{1}{2}y^2-\frac{1}{3}y\right)\left(12y+\frac{3}{5}\right)$

5. $\left(\frac{2}{5}x-\frac{3}{4}y^2\right)\left(\frac{2}{5}x+\frac{3}{4}y^2\right)$

6. $(2x-9)^2$

7. $(2x-3)(4x^2+5x-7)$

8. $(x+5)(x^2+3x+2)$

9. $(2x+5)^3$

10. $(x^2+8x+3)(2x^2-x+1)$